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A STUDY OF PROPERTIES RELATED WITH PRODUCTS OF DIFFERENT CONJUGACY CLASSES

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ABSTRACT

In physics, group theory serves as a fundamental tool for describing the symmetries and conservation laws governing the behavior of particles, fields, and physical systems. Symmetry principles based on group theory, such as the principles of relativity and gauge symmetry, underlie the formulation of fundamental theories in physics, including classical mechanics, quantum mechanics, electromagnetism, and particle physics. Group representations and symmetry operations play a crucial role in quantum mechanics, where they describe the behavior of wave functions, operators, and observables under rotations, translations, and other transformations. In quantum field theory, group theory is used to classify particle states, analyze scattering processes, and formulate gauge theories that describe fundamental interactions between particles. In chemistry, group theory provides a systematic approach to analyzing the symmetries and properties of molecules, crystals, and chemical reactions. The symmetry of molecules is described by molecular point groups, which are classified according to the symmetries of their molecular geometry and electronic structure. Group theory allows chemists to predict the spectroscopic properties, vibrational modes, and electronic transitions of molecules based on their symmetry properties, leading to insights into their stability, reactivity, and optical behavior. Moreover, group theory is essential for understanding crystallography, where it is used to classify crystal structures, determine lattice symmetries, and analyze diffraction patterns. Furthermore, group theory has applications beyond mathematics, physics, and chemistry, extending into fields such as computer science, engineering, and cryptography. In computer science, group theory is used in cryptography to develop secure encryption algorithms based on mathematical group operations, such as modular arithmetic and elliptic curve cryptography. In engineering, group theory finds

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applications in signal processing, control theory, and image recognition, where it is used to analyze the symmetries and patterns in signals, systems, and data sets. Moreover, group theory has applications in music theory, where it is used to analyze musical symmetries, harmonies, and compositions.

KEYWORDS: Conjugacy Classes, quantum mechanics, encryption algorithms, mathematical group operations

INTRODUCTION

Moreover, the symmetry-adapted basis provides a systematic framework for constructing basis functions that transform according to the irreducible representations of the molecule's symmetry group. This involves decomposing the molecular wave function into irreducible representations and constructing basis functions that transform according to each irreducible representation. The choice of basis functions depends on the molecular symmetry and the desired level of accuracy, with common choices including atomic orbitals, Gaussian-type orbitals, or Slater-type orbitals. By selecting appropriate basis functions that transform according to the irreducible representations of the molecule's symmetry group, the symmetry-adapted basis ensures that the quantum mechanical calculations are compatible with the molecular symmetry, leading to more accurate and reliable results.

Furthermore, the symmetry-adapted basis facilitates the analysis and interpretation of quantum mechanical calculations by providing a natural framework for understanding the symmetry properties of molecular systems. The irreducible representations of the molecule's symmetry group describe the distinct ways in which the molecular wave function transforms under symmetry operations, leading to symmetry labels that characterize the molecular orbitals, electronic states, and vibrational modes. By analyzing the symmetry properties of the molecular wave function, researchers can gain insights into the electronic structure, chemical bonding, and spectroscopic properties of molecules, as well as predict their reactivity, stability, and behavior in chemical reactions.

Additionally, the symmetry-adapted basis is instrumental in solving quantum mechanical problems involving molecular vibrations, rotations, and electronic transitions. In vibrational spectroscopy, for example, the symmetry-adapted basis

allows for the efficient calculation of vibrational modes and selection rules by exploiting the symmetry properties of the molecular potential energy surface. In rotational spectroscopy, the symmetry-adapted basis enables the systematic analysis of rotational states and selection rules, leading to the prediction and interpretation of rotational spectra. In electronic spectroscopy, the symmetry-adapted basis facilitates the calculation of electronic transition energies and oscillator strengths by constraining the electronic wave function to transform according to the irreducible representations of the molecular symmetry group.

Moreover, the symmetry-adapted basis plays a crucial role in computational chemistry methods such as Hartree-Fock theory, density functional theory, and many-body perturbation theory. In these methods, the molecular wave function is expanded in terms of a basis set, and the electronic structure and properties of the molecule are determined by solving the Schrödinger equation within this basis set. By using a symmetry-adapted basis, researchers can significantly reduce the computational cost of these methods and obtain more accurate results, particularly for large or complex molecular systems. The symmetry-adapted basis also allows for the systematic improvement of basis sets by adding higher-order basis functions that capture additional correlation effects or polarization effects, leading to higher accuracy in quantum mechanical calculations.

The symmetry-adapted basis represents a powerful and versatile tool in quantum chemistry and computational chemistry, enabling the efficient and accurate simulation of molecular systems. By exploiting the symmetry properties of molecular systems, the symmetry-adapted basis reduces the computational cost of quantum mechanical calculations and provides a systematic framework for understanding and analyzing the electronic structure, chemical bonding, and spectroscopic properties of molecules. Whether in vibrational spectroscopy, rotational spectroscopy, electronic spectroscopy, or computational chemistry methods, the symmetry-adapted basis plays a central role in advancing our understanding of molecular systems and their behavior in diverse chemical and physical environments.

BACKGROUND OF GROUP THEORY AS LINEAR ALGEBRA

Group theory has a close relationship with linear algebra, and many concepts and

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techniques from linear algebra can be applied to the study of groups. Here are some ways in which group theory can be viewed through the lens of linear algebra:

- Matrix Groups: Groups can be represented as collections of matrices with specific algebraic properties. For example, the general linear group GL(n, F) consists of all invertible n x n matrices over a field F. The special linear group SL(n, F) consists of all n x n matrices with determinant 1. These matrix groups have natural connections to linear transformations and can be studied using linear algebra techniques.

- Group Representations: A group representation is a homomorphism that maps group elements to invertible matrices. The representation associates each group element with a matrix, and the group operation is preserved through matrix multiplication. Group representations allow us to study groups by analyzing the corresponding matrices and their properties. Representation theory is a branch of mathematics that explores the properties and applications of group representations.

- Character Theory: Character theory is a powerful tool in the study of group representations. The character of a group representation is a function that assigns each group element a complex number based on the trace of the corresponding matrix. The characters' capture important information about the representation and provide insights into the group's structure and properties.

- Subspaces and Invariant Subspaces: In linear algebra, subspaces are subsets of a vector space that are closed under addition and scalar multiplication. Similarly, in group theory, subgroups are subsets of a group that are closed under the group operation. In both cases, the concept of closure is fundamental and allows for the study of the internal structure of the space or group. Invariant subspaces in linear algebra correspond to subgroups in group theory that are preserved under certain group actions.

- Eigenvalues and Eigenvectors: Eigenvalues and eigenvectors play a central role in linear algebra,

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representing the characteristic properties of linear transformations. In the context of group theory, eigenvalues and eigenvectors can be generalized to group actions. The study of eigenvalues and eigenvectors in group actions provides insights into the symmetries and invariant properties of the group.

- Group Rings and Modules: Group rings and modules are algebraic structures that combine the properties of groups and vector spaces. A group ring is an algebraic structure formed by formal linear combinations of group elements, with coefficients from a ring. Group modules generalize the notion of vector spaces, allowing for the action of a group on a module. The study of group rings and modules connects group theory to linear algebraic structures.

- Symmetric and Alternating Groups: The symmetric group, denoted by Sym(n), consists of all permutations of n elements. It has connections to permutation matrices in linear algebra. The alternating group, denoted by Alt(n), is a subgroup of Sym(n) that consists of even permutations. The properties and structures of symmetric and alternating groups can be analyzed using linear algebraic techniques.

- Orthogonal and Unitary Groups: The orthogonal group O(n) consists of all n x n orthogonal matrices, which preserve distances and angles. The unitary group U(n) consists of all n x n unitary matrices, which preserve inner products and lengths. These groups have connections to orthogonal and unitary transformations in linear algebra and provide a geometric perspective on group theory.

VARIOUS NOTIONS OF CONJUGACY

In group theory, the concept of conjugacy is central to understanding the relationship between elements within a group. Conjugacy arises from the operation of conjugation, which involves transforming an element of the group by applying a similarity transformation using another element. There are several notions of conjugacy that are commonly discussed in the context of groups. Let's explore these notions in detail:

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- Conjugacy of Elements: Two elements a and b in a group G are said to be conjugate if there exists an element g in G such that b = gag^(-1). In other words, a and b are conjugate if they can be transformed into each other by a similarity transformation involving g. The element g is often referred to as the conjugating element. Conjugate elements have the same cycle structure in permutation groups and share many properties and characteristics.

- Conjugacy Classes: The set of all elements in a group that are conjugate to a particular element a is called the conjugacy class of a. It is denoted by [a] or Cl(a). Mathematically, the conjugacy class of a is defined as [a] = {gag^(-1) | g ∈ G}. Conjugacy classes partition the group into subsets of elements that are related by conjugation. Each conjugacy class represents a distinct equivalence class under conjugation.

- Centralizer and Centralizers: The centralizer of an element a in a group G, denoted by C(a), is the subset of G consisting of all elements that commute with a. Formally, C(a) = {g ∈ G | gag^(-1) = a}. In other words, the centralizer of a contains all elements in G that leave a fixed when conjugated by them. The centralizer is itself a subgroup of G. The centralizer of an element provides important information about its conjugacy class and the normality of subgroups.

- Normalizer and Normalizers: The normalizer of a subgroup H in a group G, denoted by N(H), is the subset of G consisting of all elements that normalize H. An element g in G normalizes H if and only if gHg^(-1) = H, which means that conjugating H by g results in H itself. The normalizer N(H) is the largest subgroup of G in which H is a normal subgroup. Normalizers play a significant role in studying normal subgroups and factor groups.

- Conjugacy and Normal Subgroups: A subgroup H of a group G is said to be a normal subgroup if it is invariant under conjugation by all elements of G. In other words, for every g in G and h in H, the element

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ghg^(-1) is also in H. Normal subgroups form equivalence classes under conjugacy and have important properties, such as being kernels of group homomorphisms and allowing the formation of factor groups.

- Conjugacy and Group Actions: Conjugation can be viewed as a group action within a group. Given a group G, the conjugation action of G on itself is defined as the mapping (g, x) ↦ gxg^(-1) for g, x in G. This action exhibits properties similar to other group actions, such as associativity and the existence of an identity element. Conjugacy classes can be seen as the orbits under this group action.

The notion of conjugacy and its associated concepts provide powerful tools for understanding the structure and properties of groups. They allow for the classification of elements based on their transformation properties, and they reveal important connections between different elements and subgroups within a group. Conjugacy classes and related concepts have widespread applications in various branches of mathematics, such as representation theory, character theory

If G is a group and for any a, b ∈ G there exists g ∈ G such that $a = gbg^{-1}$ then we say that a is conjugate to b. Also $a = gbg^{-1}$ is equivalent to ag = gb. This motivates to define the following in semigroups.

Definition: Let S be a semigroup and a, b ∈ S , then a ∼l b if and only if there exists g ∈ S 1 such that ag = gb.

This notion of conjugacy is called as l-notion of conjugacy. This relation is always reflexive and transitive but it is not symmetric in general in an arbitrary semigroup. The following example will illustrate this fact.

Example: Let X = {1, 2}, then

$$\mathcal{T}(X) = \left\{ I = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}, f = \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}, g = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}, h = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \right\}$$

, having the Cayley table as:

∘	I	f	g	h
I	I	f	g	h
f	f	f	f	f
g	g	g	g	g
h	h	g	f	I

Here, I ◦ f = f ◦ h, this implies I∼l h. But, h /l I as there does not exists any k ∈ T (X) such that h ◦ k = k ◦ I.

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Remark: The relation ∼l gets reduced to the universal relation in a semigroup S containing zero. Since for any a, b ∈ S , there exists 0 in S 1 such that a0 = 0b, this implies a ∼l b.

Again If G is a group and for any a, b ∈ G, a = gbg−1 if and only if a = uv and b = vu (u = g, v = bg^{-1}). This motivates to define the following.

Definition: Let S be a semigroup and a, b ∈ S, then a ∼p b if and only if there exists u, v ∈ S 1 such that a = uv and b = vu.

This relation is called as p-notion of conjugacy in semigroups and on an arbitrary semi- group S , ∼p⊆∼l and as seen in case of ∼l relation, the relation ∼p also faces the problem as it is not transitive in general. However it is an equivalence relation on a free semigroups as proved in the following.

Proposition: The relation ∼p is an equivalence relation on a free Semigroup S.

Proof:

i. Reflexivity: This follows for any a, b ∈ S by taking g = h = 1.

ii. Symmetry: For any a, b ∈ S , let a ∼p b, then there exists u, v ∈ S 1 such that a = uv and b = vu, this implies b = vu and a = uv. Thus b ∼p a and so ∼p is symmetric.

iii. Transitivity: For any a, b, c ∈ S such that a ∼p b and b ∼p c, then there exists u1, v1 and u2, v2 ∈ S 1 such that

$$a = u_1v_1,\ b = v_1u_1 \text{ and } b = u_2v_2, c = v_2$$

We show that a ∼p c. Let v1 = $x_1 x_2 \cdots x_{n1}$, $u_1 = y_1y_2 \cdots ym_1$, $v_2 = z_1z_2 \cdots zn_2$ and $u_2 = t_1t_2 \cdots tm_2$. Since $v_1u_1 = u_2v_2$, therefore $x_1 x_2 \cdots xn_1\ y_1y_2 \cdots ym_1 = t_1t_2 \cdots tm_2\ z_1z_2 \cdots zn_2$. From equality of two words, we have n1 + m1 = m2 + n2. We have the following cases.

Case (a): If $n_1 = m_2$ then $m_1 = n_2$. This implies $x_i = t_i$ for all i = 1, 2, · · ·, n_1 and $y_i = z_i$ for all i = 1, 2, · · ·m_1. This implies $v_1 = u_2$, $v_2 = u_1$. So $a = u_1v_1$, $c = v_2u_2 = u_1v_1$. This implies a = c and thus by reflexivity a ∼p c. Hence ∼p is transitive.

Case(b): If $n_1 < m_2$ [$n_1 > m_2$]. We prove the case when n1 < m2, the other case follow on the similar lines. We have $m_2 = n_1 + k$ for some integer k. So we have

$$x_i = t_i \text{ for all } i = 1, 2, \cdots, n_1.$$
$$y_j = t_{n_1+j} \text{ for all } j = 1, 2, \cdots, k.$$
$$\text{and } y_{k+t} = z_t \text{ for all } t = 1, 2, \cdots, n_2.$$

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With these notations, we have $v_1 = x_1 x_2 \cdots x_{n_1} = t_1 t_2 \cdots t_{n_1}$. So,

$$v_1 y_1 y_2 \cdots y_k = v_1 t_{n_1+1} t_{n_1+2} \cdots t_{n_1+k}$$
$$= t_1 t_2 \cdots t_{n_1} t_{n_1+1} t_{n_1+2} \cdots t_n$$
$$= u_2.$$

Again, $u_1 = y_1 y_2 \cdots y_k y_{k+1} \cdots y_{m1} = t_{n1}+1 t_{n1+2} \cdots t_{n1+k} z_1 z_2 z_{n2} = t_{n1+1} t_{n1+2} \cdots t_{n1+k} v_2$.

Now, $a = u_1 v_1 = t_{n1+1}\, t_{n1+2} \ldots t_{n1+k} v_2 v_1$ and $c = v_2 u_2 = v_2 v_1 t_{n1+1} t_{n1+2} \ldots t_{n1+k}$. So, $a \sim pc$. Hence, $\sim p$ is transitive.

AUTOMORPHISM AND INFINITE PARMUTAIONS GROUP

We consider the following for the sets Ω, K and Φ, where Φ is the set of constraints of members of G to members of K, and the group G is the group of permutations of the infinite set.

1. Under the operation of G, K is an orbit of non-empty subsets of Ω.
2. K ∪ {Φ, Ω} under the normal Operations ∪ and ∩, it is a Boolean algebra.
3. The union of any three elements of K is countable and disjoint.
4. If X ∈ K and σ ∈ G and X ∩ supp σ ≠ Φ there is Y ⊆ X in K such that Y ∩ σ Y = Φ.
5. If X, X_n, Y, Y_n ∈ K are such that X= $U_{n\in\omega} X_n$ and y= $U_n \in {}_\omega Y_n$ are disjoint unions and $\emptyset_n \in \Phi$ has domain X_n and range Y_n for each n, then $U_n \in {}_\omega \emptyset_n \in \Phi$.
6. If Ω = $X_1 \cup X_2 = Y_1 \cup Y_2$ are disjoint unions and $\emptyset_i$ has domain X_i and range Y_i(i= 1,2,....) then $\emptyset_1 \cup \emptyset_2 \in G$.

Theorem

Assume G, Ω and K are the same as in (i). Members σ, τ, of G that are not identical to one another are conjugates σ_1, σ_2, σ_3 of σ such that $\tau = \sigma_1^{-1} \sigma_2 \sigma_3$.

To prove the above theorem, we first of all prove the following lemma.

LEMMA

Suppose that σ, τ ∈ G and X ∈ K are such that X ∩ τX= ∅ and σx = τx for all x ∈ X. Then there is θ ∈ G such that $\tau\theta^{-1} \sigma^{-1} \theta \in \Sigma \subseteq G$.

Proof.

from (4.l)(iii) we find that any T∈K has a proper subset S lying in K, and from (4.l)(ii)

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that T-S also lies in K. There we have the following two cases.

Case 1. σ -1 τ is also the identity on τX. Let Y be a proper subset of τX lying in K. By (3.l) (iii) we may write Y as the disjoint union Un∈ wYn of members of K and we also let Zn = Yn for each n.

Case 2. σ -1 τ is not the identity on τX. Then by (3.l)(iv) (and the fact that by (i), τX ∈ K) there is Y ⊆ τX such that Y ∩ σ-1 τY =∅. Using (iii) to decrease Y if necessary we may assume that τX ⊈ Y ∪ σ -1 τY. By (ii) and (iii) Y may be written as the disjoint union U ∪ V ∪ W of three members of K and by (ii), U ∪ W, V ∪ W∈ K. By (iii) we may write

U = Un∈w Un and V = Un∈w Vn where Un, Vn ∈ K and the unions are disjoint. We let Y0 = U ∪ W, Yn+1 = Vn, Z0 = V ∪ W, Zn+1 = Un. Thus for each n, Yn,Zn∈ K. Also Y0 ∪ Z0=Un∈w Yn=Un∈wZn=Y and Y0 ∩ Z0= W ∈ K.

STABILIZERS OF FINITE SETS

For the applications of this work we need to know that the stabilizers of finite subsets in certain infinite permutation groups are simple, or at least not too far from being simple. In the general case, for a group G which need not be simple we may introduce the quasi-order ≼ by defining τ ≼ σ to mean that τ lies in the normal subgroup generated by σ and ask, when this occurs, for the minimum n such that τ may be written as the product of n conjugates of σ or σ-1. We spend the most time addressing this question for the stabilizer of a single point in the homeomorphism group of Q, Ir, or C. The situation for arbitrary finite sets follows quite easily from this.

Continuity is harder to arrange since the complement of a singleton is open but not closed.

Let G be a permutation group on the set Ω. If these are understood we denote by K(a) and K(A) the stabilizer and point wise stabilizer, respectively, of Q ∈ Ω and A ⊆ Ω . The proof of the “three conjugates” result for K(a) is modelled closely on that of Theorem 3.1.2 but we require additional terminology and a number of preliminary results. Let us say that X⊆ Ω abuts a (where a⊆ Ω and Ω = Q, Ir, or C) if X is open, a ∉ X, and X= XU {a}. We let L(a) be the subgroup of K(a) comprising all σ ∈ K(a) which fix pointwise a neighbourhood of a. Then L(a) is clearly a proper non-trivial normal subgroup of K(a). Theorem 3.2.13 will show that it is the only such.

1. Lemma

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Any two non-empty clopen subsets of Ω are homeomorphic.

Proof.

This has already been used implicitly in fact (for example, in the proof of Theorem 3.8) and it is remarked on in [59], but we outline a short proof here because of its important role in what follows. For Ω = Q it is immediate from Sierpinski's Theorem. In fact for Ω = Q or Ir one may show that any two non-empty open subsets X and Y of Ω are homeomorphic. This is because each of X and Y may be written as the disjoint union of ℵ0 clopen intervals, since R - Ω is dense in R. For C we deduce by compactness that any two non-empty clopen sets X and Y are finite disjoint unions of basic clopen intervals (where the "basic clopen" intervals are C, C ∩ [0, ⅓], C ∩ [⅔, 1], C ∩ [0, 1/9], etc.). But in C any basic clopen interval is homeomorphic to C, and C is the disjoint union of an arbitrarily large finite number of basic clopen intervals, so the result is clear.

Now each of the spaces Q, Ir, and C is a subspace of R, hence a metric space in the natural way. If ∅ ≠ X ⊆ Ω we let d(a, X) = sup {|a – x| : x ∈ X} (which may equal ∞). (A more standard notion of the "distance" from a point to a set would perhaps be inf {|a – x| : x ∈ X }, but it is sup that we require here.)

2. Lemma

Suppose that X abuts a. Then X may be written as a countable disjoint union of non-empty clopen sets Xn, such that d(a, Xn) ≤ l/n for each n.

Proof.

Let (Yn)n∈ω be a sequence of clopen sets in Ω each containing a such that Ω = Y0 ⊇ Y1 ⊇ Y2 ⊇ .....= and d(a, Yn) ≤ l/n. Then for each n, Xn' = X∩ (Yn – Yn+1) is clopen and as a ∈ X, X=Un∈ωXn'. Since a∈X, {n : Xn' ≠ ∅} is infinite, and we let (Xn) enumerate the non-empty Xn' in increasing order.

3. LEMMA

Suppose that X and Y are sets abutting a. Then there is a Homeomorphism from $\bar{X}$ to $\bar{Y}$ fixing a.

Proof By Lemma 3.2.2 we may write X= Un∈ωXn, Y= Un∈ωYn where these are disjoint unions of non-empty clopen sets and d(a, Xn), d(a, Yn) ⩽ l/n. Let 8: X → Y p be defined by

$$\theta x = \begin{cases} \theta_n x \ if \ x \in X_n \ some \ n \\ x \quad if \ x = a \end{cases}$$

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where θn : Xn → Yn is a homeomorphism as provided by Lemma 3.2.1. To check continuity we suppose that xm → x as m → ∞. If x ∈ Xn for some n then as Xn is open, xm ∈ Xn eventually, so θxm → θx by continuity of θn. If x = a then as each Xn is closed, {m: xm ∈ Xn} is finite. Let xm ∈ Xn(m) (for those m such that xm ≠ a). Then n(m) → ∞ as m → ∞ (where it exists).

Since d(a, Yn) → 0, also d(a, Yn(m)) → 0 and |a – θxm| → O showing that θxm → θx as required. Similarly θ-1 is continuous.

4. Lemma

If X, Y each abut a then so does X ∪ Y.

Proof.

X ∪ Y is open, a ∉ X ∪ Y, and X ∪ Y = X ∪ Y = X ∪ Y ∪ {a}.

5. Lemma

Zf X abuts a and a ∈ Ω - X- (a} then Ω - X- {a} abuts a.

Proof.

Ω – X - {a} is open since it equals Ω – X, a ∈ Ω - X- {a}, and Ω - X- {a} ⊆ Ω - X = Ω - X so that in view of the assumption, Ω - X- {a} ⊆ (Ω - X- {a}) ∪ (a} ⊆ Ω - X- {a} showing Ω - X- {a} = (Ω - X- {a}) ∪{a}.

CONCLUSION

The class equation of a group provides information about the number of elements in each conjugacy class. It is obtained by partitioning the group into its distinct conjugacy classes. The class equation can give insights into the structure of a group and its subgroups. Studying the products of conjugacy classes can provide information about the structure and subgroups of a group. The product of two conjugacy classes can be related to the structure of the group, such as its normal subgroups, quotient groups, or the behavior of elements under specific group operations. In conclusion, conjugacy classes and their products are important concepts in group theory. The study of conjugacy classes and their products helps to understand the structure and properties of groups, including their subgroups, quotient groups, and group operations. The class equation provides a useful tool to analyze the number of elements in each conjugacy class. The centralizer and normalizer play a crucial role in understanding conjugacy classes and their interactions within a group.

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