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STUDYING ABOUT THE GENERATING FUNCTIONS

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ABSTRACT

Here, for the first time, generating functions are shown in their generic form. We may use the generating function to obtain the coefficients of a number of different polynomials. In addition, we compute the nth term of the polynomial. Special functions, and more especially hypergeometric functions and polynomials in one or more variables, are frequently needed for addressing problems in physics, engineering, statistics, and operations research. In the theory of special functions, some authors have recently focused on generation functions, summations, and transformations formulas. It is via the use of transformations, finite sum characteristics, and generating functions that the study of special functions may progress. For special functions and polynomials in one, two, or more variables, this dissertation covers certain classes of generating functions, such as linear, bilinear, bilateral, double, and multiple generating functions. The series rearrangement method, integral operator approaches, Nishimoto's fractional calculus, and group theoretic techniques can all be used to obtain these generating functions.

Keywords: - Function, Generic, Polynomials, and Formulas.

I. INTRODUCTION

It is well-established that generating functions are vital for examining many intriguing and maybe beneficial properties of the sequences they create. These activities are in charge of manufacturing. These sequences are made feasible by the generating functions. Alternatively, generating functions may be used to turn differential equations describing discrete-time signals and systems into algebraic equations. The construction of functions has yet another utility here. This allows the simplification of a broad variety of problems in operations research, mathematics, and other practical areas that demand sequential fractional-order difference operators. Several other fields of applied research (such as, for instance, queuing theory and related stochastic processes) may also benefit from similar qualities. This method is a means of making effective use of generating functions. $\{f_n\}_{n=0}^{\infty}$ One of the processes in adapting Darboux's method is analyzing the asymptotic behavior of the new sequence that has been generated. This is a very significant step along the way.

Furthermore, if a series has a generating function, then follows that $\{f_n\}_{n=0}^{\infty}$ of numerical values or mathematical operations might be beneficial in identifying $\sum_{n=0}^{\infty} f_n$ using tactics of whole ability, as the ones created by Abel and Cesaro and a slew of other folks. According to Lando's work and the supporting materials, the language of generating functions is now the preferred way of communication in the field of combinatory analysis. The study of generating functions just requires a knowledge with calculus and algebra, and not a vast range of other specialized areas of mathematics. The widespread usage of generating functions in fields like mathematics and computer science suggests they might potentially be an effective instrument for furthering the field of mathematics education. This is owing to the essential role that generating functions play in these processes.

II. GENERATING FUNCTIONS

In the next portion of this article, we will make an attempt to describe the general form of the generating function. The polynomial functions and coefficients may be extracted as well, providing us with two crucial building blocks for creating specialized functions.

$$G(x,t) = \sum_{n=0}^{\infty} F_n(x)t^n, (8)$$

In order to derive the Fn (x) using the equation, we get here

$$F_n(x) = \frac{1}{n!}\frac{\partial^n G(x,t)}{\partial t^n} \mid t = 0 \ (10)$$

where the equation is a theoretical scaffolding for the procedure of summing up the individual polynomials. The next step is to determine the coefficients of an, which play a critical role in the special function. To locate this, see the preceding sentence's explanation. This is the next logical step once the previous one has been completed. Presented below for your perusal is the updated polynomial Fn(x). Presently, it seems like this:

$$F_n(x) = \sum_{n=0}^{\infty} a_n x^n \ (11)$$

Also we have

$$F_n(x) = \sum_{n=0} \frac{x^n}{n!}\frac{\partial^n F_n(x)}{\partial x^n} \ (12)$$

Finally, the equation allows us to obtain the following expression:

$$a_n = \frac{1}{n!^2}\frac{\partial^{2n} G_n}{\partial x^n \partial t^n}(x=0, t=0) = \frac{1}{n!}\frac{\partial^n F_n(x)}{\partial x^n}\ _{(x=0)}$$

III. SOME POLYNOMIALS DEFINED BY GENERATING FUNCTIONS AND DIFFERENTIAL EQUATIONS

Research on the characteristics of polynomials is necessary, and the creation of new polynomials via the use of unconventional generating functions is not only intriguing but also necessary. The first person to characterise polynomials by making use of the generating function was Humbert (1921) $\Pi_{n,m}^{\nu}(x)$, n $= 0,1,2,\ldots,$ might be any value from.

$$(1 - mtx + t^m)^{-\nu} = \sum_{n=0}^{\infty} \Pi_{n,m}^{\nu}(x)t^n. (14)$$

Gould (1965) provided the generalization of the Humbert polynomial of degree n and was also responsible for the name. Using difference operators, Milovanovi and Djordjevic performed 1987 research that led to the formulation of a differential equation for the function. The generalized Hermite polynomials were defined by Lahiri (1971) through the generating function.

$H_{n,m,v}(x), n = 0,1,2,\ldots,$ where n may range anywhere from.

$$\exp(vtx - t^m) = \sum_{n=0}^{\infty} H_{n,m,v}(x)\frac{t^n}{n!} (15)$$

The other extension of Hermite polynomials by the generating function was introduced by Gould and Hopper (1962).

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$x^{-a}(x-t)^{a}\exp\left(p\left(x^{r}-(x-t)^{r}\right)\right)$

The analysis being performed is identical to that described by Bell (1934), with the setting of a=0. We spoke about how there could be room for polynomials in Suzuki $Q_n(x;k,v), n=0,1,2,...,$ by making use of the following generating function, which has qualities that are comparable to those of the Humbert polynomials.

$$(1-2tx+t^{k})^{-v}=\sum_{n=0}^{\infty} Q_n(x;k,v)t^{n},$$

where k is an integer such that $k \geq 2$ and v is a positive real number. Note that

$\Pi_{n.m}^{v}(x)=Q_n(mx/2;m,v)$

the polynomial $Q_n(x;k,v)$ is not a completely original thought. But we explained the function using a differential equation. An explicit expression is given instead, and it does not include any disjunction operators. After setting x = 0 and completing some further work, we obtained the universal solution to the differential equation. Having fulfilled that requirement This made it possible for us toWe examine the potential of using the generating function to construct an extension of the Hermite polynomials in the work named Dobashi (2014). This study just came out in the public domain.

$$\exp\left(t^{k}x-t^{k+j}\right)=\sum_{n=0}^{\infty} R_n(x;k,j)t^{n}, (16)$$

Taking it for granted that both k and j are positive integers. And the results that we obtained were similar to those that were found in the instance of $Q_n(x;k,v)$ In this particular instance, the general solution that corresponds to it is written down as a linear combination of functions that are stated by the application of ${}^{2}F_{k+j-1}$ type hypergeometric functions. And the results that we obtained were comparable to those that were found in the instance of $Q_n(x;k,v)$ The differential equations for $R_n(x;k,j)$, and R n (x;k,j) are the subject of the study for this work, as well as obtaining the general solutions for those differential equations when x=0 is taken into consideration. The debate of $Q_n(x;k,v)$ can be found in, and the discussion regarding $R_n(x;k,j)$ can be found in.

IV. SOME POLYNOMIALS AND SPECIAL FUNCTIONS BY USING LIE LAPLACE TRANSFORMATION

Courant and Hilbert stumbled across the idea of special functions in 1953 while investigating the use of ordinary differential equations in physics. The study of special functions ultimately became its own field of study because of this. At the same time, Morse and Feshbach were investigating the potential applications of special functions to the study of physics-related problems. There have been advancements in the field of special functions as a direct result of this. Paul Turan claims he has evidence from a long time ago that special functions have been in use. His experience has led him to this conclusion. Euler, Legendre, Laplace, Gauss, Kummer, Riemann, and Ramanujan are only few of the well-known mathematicians of the 18th and 19th centuries who made significant contributions to the theory of special functions. Many individuals, including Riemann and Kummer, have contributed significantly to the development of the theory of special functions. The unique characteristics are still being studied for the same reasons they were studied in the past.

Their interactions with other branches of mathematics and science, including number theory, combinatorics, computer algebra, and representation theory, are only a few examples. There are further illustrations provided. Readers interested in this topic would do well to familiarize themselves with the excellent works on the subject produced by writers such as Andrews, Rao, Rose, and Rainville. Miller's research contributed to the development of Weisner's theory by forging a link between it and Schrodinger's factorization method. Miller made this discovery and contributed it to science. The following is Miller's contribution to the development of Weisner's theory. His demonstration of a connection between the theory and the work of Infield and Hull further extended its usefulness. Lie algebraic characterizations of two-variable Horn functions are an area of research that has benefited from the work of Kalnins, Onacha, and Miller. All three scientists have been investigating this topic. They do this by transforming a two-variable Horn function into a family of one-variable hyper geometric functions. As a result, an approach is designed and created for creating generating functions. The bulk of this chapter is devoted to discussing hyper geometric functions in one, two, and even more variables. Basic functions like the Gamma and Beta functions, together with their definitions and salient features, are discussed before being given as potential solutions to the problem at hand.

V. CONCLUSION

Hypergeometric functions and polynomials in one or more variables are two examples of special functions that appear often in many different types of problems in the fields of physics, engineering, statistics, and operations research. In recent years, a lot of writers have focused on the formulas for special functions' generating functions, summations, and transformations. Generating functions, finite sum properties, and transformations are all important to consider while studying special functions. This survey will also include subsets of generating functions because of their increasing significance. Special functions and polynomials in one, two, or more variables may have linear, bilinear, bilateral, double, or multiple generating functions, depending on the nature of the function or polynomial. Several techniques exist for creating such functions, such as those based on series rearrangement, integral operator methods, Nishimoto's fractional calculus, and group theory. Some hypergeometric functions and polynomials are transformed, their fractional derivative formulae are given, and their finite sum properties are discussed. Several finite sum characteristics, transformations, and generating functions for the Laguerre polynomials in two, three, four, and more than two variables are sought for, and this study aims to provide the framework for these endeavors. Finding some double generating relations for the generalized hypergeometric function is the focus of this section.

REFERENCES

1. Zudilin, Wadim. (2012). A generating function of the squares of Legendre polynomials. Bulletin of the Australian Mathematical Society.

89. 10.1017/S0004972713000233

2. SZEGO, G.: ¨ Orthogonal Polynomials (4th ed.). Amer. Math. Soc. Colloq. Publ. 23, Amer. Math. Soc., Providence, RI, 1975.
3. Srivastava, R.; Cho, N.E. Generating functions for a certain class of incomplete hypergeometric polynomials. Appl. Math. Comput. 2012, 219, 3219–3225.
4. Srivastava, H.M.; Gangopadhyay, G. The absorption bandshape function of a molecule from a thermocoherent state and some associated multilinear generating-function relationships for Laguerre polynomials. Russian J. Math. Phys. 2004, 11, 359–367.
5. Shahwan, Mohannad & Bin-Saad, Maged & Shahwan, M. & Sharif, Ameera. (2019). Generating functions for generalized Hermite polynomials associated with parabolic cylinder functions. Integral Transforms and Special Functions. 31. 10.1080/10652469.2019.1697695.
6. S. Khan, M.A. Pathan, G.Yasmin, Representation of a Lie algebra G (0, 1) and Three Variable Generalised Hermite Polynomials, Hn (x, y, z), Integral Transforms Special Functions, 13 (2002), 59-64.