International Journal For Advanced Research In Science & Technology
A peer reviewed international journal www.ijarst.in
IJARST
ISSN: 2457-0362

A STUDY OF NUMERICAL SOLUTIONS BASED ON THE WAVELET FOR DIFFERENTIAL EQUATIONS

CANDIDATE NAME= JAVID
DESIGNATION- RESEARCH SCHOLAR MONAD UNIVERSITY HAPUR U.P
GUIDE NAME = DR. HOSHIYAR
DESIGNATIO- ASSISTANT PROFESSOR MONAD UNIVERSITY HAPUR U.P

ABSTRACT

The study of numerical solutions for a variety of integral and differential equations has paid a lot of attention in recent years to the use of wavelet technology. Insight into current work to improve wavelet-based numerical solutions to a wide range of integral and differential equations is provided in this abstract. The wavelet approach makes use of the wavelet transformations mathematical framework, which provides a potent tool for breaking down complicated functions into a smaller collection of basis functions. Problems involving localized phenomena, such as abrupt gradients or singularities, are frequent in many physical and engineering systems, and may be effectively tackled with the help of this decomposition since it allows for the representation of functions at multiple resolutions and scales. The fundamental goal of this study is to improve and broaden the range of mathematical models that may benefit from wavelet-based numerical approaches. This comprises differential equations from the ordinary to the partial order, as well as integral equations like the Fredholm and Volterra equations. By taking use of wavelets' flexibility and multiscale structure, we hope to speed up convergence of numerical solutions to these equations and increase their accuracy and efficiency.

KEYWORDS: Numerical Solutions, Differential Equations, numerical solutions, differential equations.

INTRODUCTION

Integral and differential equations are only two examples of the many mathematical problems that benefit greatly from the use of numerical techniques. Numerous methods have been created throughout time to enhance the precision and performance of numerical solutions. The wavelet technique is one such approach that has received a lot of attention.

The wavelet technique is a mathematical framework for solving difficult signal processing, picture compression, and data analysis issues using wavelet analysis. However, it has been found to be quite useful in solving integral and differential equations, and its use has spread outside these domains.

Due to its capacity to capture localized characteristics and flexibility across multiple scales, wavelet-based approaches have recently garnered attention as a promising tool for numerical solutions. Wavelet-based techniques give a flexible framework that may adapt to the peculiarities of the issue at hand, unlike classic numerical methods like finite difference or finite element methods, which need a set grid or basis functions.

One of the primary benefits of the wavelet approach is that it can deal with situations involving fast changing characteristics, as well as uneven domains and boundaries. The approach successfully captures both global and local characteristics of the solution through the use of wavelet functions, which are localized in both the time and frequency domains, resulting to improved accuracy and efficiency.

Wavelet analysis may also be used for multiscale analysis, breaking down an issue into smaller subproblems at various frequencies or resolutions. This decomposition allows for the efficient modeling of the solution using a sparse set of coefficients, which in turn reduces computing costs and memory requirements by facilitating the identification of dominating features.

Numerous scientific and technological disciplines have found success in applying wavelet-based approaches to integral and differential equations. They have been used to great effect in a wide variety of fields, including fluid dynamics, electromagnetics, finance, and image processing, to name a few. Hybrid approaches that incorporate the benefits of several methodologies have been developed by fusing the wavelet approach with other numerical techniques, such as finite element methods.

HISTORICAL PERSPECTIVE OF WAVELETS

French mathematician J. Fourier, who was interested in both mathematics and mathematical physics, is credited with developing the harmonic analysis that forms the basis of Wavelet Theory in his renowned "analytique de la Chaleur". In contrast, A. Haar initially mentioned wavelets in his theory "Zur Theorie der orthogonalen functionen systeme" in 1909; this theory introduced a family of functions now known as Haar wavelets. For analyzing functions (signals) that are more locally distributed in time-domain than the harmonic functions employed in the Fourier analysis, it forms the simplest known wavelets sets. Although wavelets were first introduced in the early 1980s, they quickly gained popularity and promise in a wide range of scientific fields.

At the "Elf Aquitaine oil Company" in 1982, the French geophysical engineer J. Morlet utilized a Short Time Fourier Transform (STFT) with a Gabor window, in which a window of constant length is swept across the data, to examine changes in the frequency spectrum over time. Because high-frequency signals need small time-domain windows and low-frequency signals require long time-domain windows, he discovered a flaw in this analysis approach owing to the fixed length of the Gabor window. So, he tweaked this window by creating stretched and compressed variants of one-of-a-kind oscillating windows while maintaining the same amount of oscillations. He found that this method yielded more consistent and precise results than the STFT. Morlet wavelets were named after him because he used a stretched and translated function for analysis.

The Morlet's wavelet transform is fairly comparable to the formulism for coherent states in quantum physics, and A. Grossmann, a French theoretical physicist, immediately realized its usefulness and created an accurate inversion formula for this transform. In other words, the coefficients of a wavelet modification allow for a flawless recovery of the original signal. Furthermore, J. Morlet and A. Grossman demonstrated that the signal reconstructed from a slightly changed wavelet transformation is also somewhat affected. This is a critically significant finding, since it provides the theoretical foundation for the approaches used in wavelet-based denoising techniques. Denoising techniques often involve making small adjustments to the wavelet coefficients in an effort to reduce noise. Our focus is pinpointed

International Journal For Advanced Research In Science & Technology
A peer reviewed international journal
www.ijarst.in
IJARST
ISSN: 2457-0362

by the wavelets in the translated version. The scaled-down wavelets, on the other hand, enable us do multi-scale analyses of the signal.

FROM FOURIER TRANSFORMS TO WAVELET TRANSFORMS

Fourier Transform

Definition 1.1 Let $\Omega \subset R^n$ and $p \in R^+$. We denote Lp (Ω) the class of all measurable function f , defined on Ω as

$$\int_{\Omega} |f(t)|^p dt < \infty,$$

with finite L^p -norm defined by

$$\|f\|_p = \left(\int_{\Omega} |f(t)|^p dt\right)^{1/p}, \quad when \quad 1 \leq p < \infty,$$

and

$$\|f\|_p = \operatorname*{ess\,sup}_{-\infty<t<\infty} |f(t)|, \quad when \quad p = \infty.$$

Definition 1.2 For f, g $\in L^2$ (R), we define the inner product

$$\langle f, g \rangle = \int_{-\infty}^{\infty} f(t)\,\overline{g(t)}\,dt.$$

Definition 1.3 The operator T stands for an integral transform, which is an operator on the $\Omega \subset R^n$ which is defined by

$$(Tf)x = \int_{\Omega} K(t,x)\,f(t)\,dt,$$

and the transform kernel is denoted by K.

The transform's characteristics are determined by the kernel.

Definition 1.4 Let f $\in L^1$ (R), Consequently, the expression for f's Fourier transform is

$$\hat{f}(\omega) = \mathscr{F}[f(t)] = \int_{-\infty}^{\infty} f(t)\,e^{-i\omega t}\,dt,$$

where F is called the Fourier transform operator.

Definition 1.5 Let ˆf $\in L^1$ (R) is the Fourier transform, then define f of f to be the inverse of f using

$$\mathscr{F}^{-1}(\hat{f}(\omega)) = f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{f}(\omega)\,e^{it\omega}\,d\omega,$$

where F^{-1} is the inverse Fourier transform operator

Note: The spectrum behavior of the signal f is described by the Fourier transform, and the inverse Fourier transform returns the data from the frequency domain to the time domain.

Definition 1.6 Let f,g $\in$ L 1 (R), then (f g), the convolution of f and g, is defined as

International Journal For Advanced Research In Science & Technology
A peer reviewed international journal
www.ijarst.in
IJARST
ISSN: 2457-0362

$$(f*g)(x)=\int_{-\infty}^{\infty} f(x-t)\,g(t)\,dt.$$

MULTIRESOLUTION ANALYSIS (MRA)

In the framework of wavelet analysis, S. Mallat and Y. Meyer established a multiresolution analysis (MRA) in 1986. The pyramid approach of image processing, first proposed by J. L. Crowley in 1982, and the theory of differential equations in microlocal analysis are its ancestors. Indeed, multiresolution analysis is at the heart of every attempt to develop a wavelet foundation, therefore this novel and astonishing proposal is concerned with a generic formalism for doing so. With MRA, the same sized window is used to analyze data from objects of varying sizes, with the smaller items analyzed at a higher resolution and the larger ones at a lower one. Mallat's groundbreaking research from 1989 has been the impetus for several innovations in wavelet analysis and its many practical applications across a broad range of mathematical disciplines and industries.

Multiresolution analysis is a branch of mathematics that aims to express a function f as the limit of consecutive approximations, where each succeeding approximation is a more precise version of the original function f. These consecutive approximations map to ever finer levels of detail. The ability to get the information we need at the size and location of our choosing is made possible by the multiresolution representation of data such as audio, video, pictures, etc. In this way, multiresolution analysis may be seen as a systematic method for building orthogonal wavelet bases according to a predetermined protocol. The foundation of the MRA is concerned with breaking down the whole area into smaller, more manageable chunks. $V_j \subset V_{j+1}$, so that the space V_{j+1} contains all the scaled-down versions of the V_j . This effectively implies that each function is decomposed into scale-dependent components, with each subspace containing a unique scale-dependent component of the original function f.

Definition 1.12 (Analysis at Multiple Scales) Definition of Multiresolution Analysis $V_j : j \in \mathbb{Z}$ of $L^2(\mathbb{R})$ with closed embedded subspaces that adhere to the following conditions

(i) Monotonicity: $V_j \subset V_{j+1}$, *for every* $j \in \mathbb{Z}$

(ii) Density: $\overline{\bigcup_{j\in\mathbb{Z}} V_j} = L^2(\mathbb{R})$

(iii) Separation: $\bigcap_{j\in\mathbb{Z}} V_j = \{0\}$

(iv) Scaling: $f(t) \in V_j \Leftrightarrow f(2t) \in V_{j+1}$

(v) Orthonormal basis: $\exists$ *a scaling function (father wavelet)* $\phi \in V_0$ *i.e* $\{\phi_{0,k} = \phi(t-k) :$ $k \in \mathbb{Z}\}$ *is a orthonormal basis for* V_0.

Remark 1.1

(a) Condition (i) to (iii) mean that every function in $L^2(\mathbb{R})$ can be approximated by elements of the subspaces V_j , and as j approaches ∞, the precision of approximation increases.

(b) Conditions (iv) express the invariance of the system of subspaces {Vj} with respect to the dilation operators.
(c) Condition (v) can be rephrased for each j ∈ Z that the system {2 j/2ϕ(2 j t −k)}k∈Z is an orthonormal basis of Vj .
(d) For a given MRA {Vj} in L 2 (R) with scaling function ϕ, a wavelet is obtained in the following manner. Let the subspace Wj of L 2 (R) be defined by the condition

$$V_j + W_j = V_{j+1}, \quad V_j \perp W_j \quad \text{for all } j$$

$$V_{j+1} = J_j(V_0 \oplus W_0) = J_j(V_0) \oplus J_j(W_0) = V_j \oplus J_j(W_0),$$

where J_j (for an integer j, J_j is defined as $J_j(f(t)) = f(2^j t)$ for all $f \in L^2(\mathbb{R})$) is an isometry, $J_j(V_1) = V_{j+1}$.

$$V_m = \bigoplus_{j \geq m+1} W_j.$$

This gives

$$W_j = J_j(W_0) \quad \text{for all } j \in \mathbb{Z}.$$

From condition (i) to (iii), we obtain an orthogonal decomposition

$$L^2(\mathbb{R}) = \oplus \sum_{j \in \mathbb{Z}} W_j = W_1 \oplus W_2 \oplus W_3 \oplus \ldots.$$

$$= \bigoplus_{j \in \mathbb{Z}} W_j.$$

Let $\psi \in W_0$ be such that $\{\psi(t-m)\}_{m \in \mathbb{Z}}$ orthonormal basis in W_0, then $\{\psi_{j,k} = 2^{j/2}\psi(2^j . - m) : m \in \mathbb{Z}\}$ is an orthonormal basis of W_j. This function is a wavelet. Let $\phi(t) = \sum_{n \in \mathbb{Z}} c_n \phi(2t-n)$, where c_n is an appropriate constant, and then $\psi(t) = \sum_{n \in \mathbb{Z}}(-1)c_{n+1}\phi(2t + n)$.

(e) It's worth noting that not everyone uses the norm of naming subspaces in ascending order from "Vi" to "Vj." Definitions often use a diminishing succession of subspaces Vj. The end outcome is the same either way.
Since V0 ⊂ V1, It is possible to express any function in V0 in terms of the basic functions of V1. Most notably, ϕ(t) = ϕ0,0 ∈ V0 and hence

$$\phi(t) = \sum_{k=-\infty}^{\infty} a_k \phi_{1,k}(t) = \sqrt{2} \sum_{k=-\infty}^{\infty} a_k \phi(2t-k).$$

where

$$a_k = \int_{-\infty}^{\infty} \phi(t)\phi_{1,k}(t)dt. \qquad (1.17)33$$

Only a limited number of ak's will be nonzero for compactly supported scaling functions, and we have [43]

$$\phi(t) = \sqrt{2} \sum_{k=0}^{D-1} a_k \phi(2t-k).$$

Eq. termed the dilation equation, is a cornerstone of wavelet theory. The wavelet genus D is an even positive integer, and the filter coefficients are the values a0, a1,....., aD1.

CONCLUSION

Differential equations describe the connection between these variables and their derivatives. Mathematical, societal, and scientific problems all eventually include differential equations. For example, in ecology (population modeling, monotonicity of population), geology (weather forecast modeling, detection of seismic waves below ground), biology (infectious diseases, genetic variation), chemistry (reaction rates), economics (stock exchange, market rate according to sale), and a wide variety of engineering applications (diffusion), differential equations are commonly encountered in connection with a wide variety of problems. Some examples of generic differential equations are the problems of determining the velocity of a projectile, the charge or current in an electric circuit, the development of a population, and the conduction of heat in a rod. Some kind of categorization is obviously required. Differential equations are often sorted into two groups: those with one independent variable and those with several variables.

REFERENCES

1. S. Abbasbandy, Numerical solution of integral equation: Homotopy perturbation method and Adomian decomposition method. Appl. Math. Comput., 173 (2006), 493-500.
2. N. H. Abel., Auflo¨sung einer mechanischen Aufgabe. J. f u¨r reine und angew. Math., 1 (1826), 153-157.
3. N. H. Abel., Solution de quelques proble`mes a` l'aide d'inte`grales de`fnies. Oeuvres Comple`tes, 1(1881), 16-18.
4. M.J. Ablowitz, B.M.Herbst and C.Schober, Constance on the numerical solution of the sine- Gordon equation. I: Integrable discretizations and homoclinic manifolds. J.Comput.Phys., 126 (1996), 299-314.
5. M.J. Ablowitz, B.M. Herbst and C.M. Schober, On the numerical solution of the sine-Gordon equation. II. Performance of numerical schemes. J. Comput. Phys. 131 (1997), 354-367.
6. M.J. Ablowitz, B.M.Herbst and C.Schober. Constance on the numerical solution of the sine- Gordon equation. I: Integrable discretizations and homoclinic manifolds. J.Comput.Phys., 126 (1996), 299-314.
7. H. Adibi and P. Assari, On the numerical solution of weakly singular Fredholm integral equations of the second kind using Legendre wavelets. J. Vib. Control., 17 (2011), 689-698.
8. G. Adomian, A review of the decomposition method and some recent results for nonlinear equa- tion. Math. Comput. Model., 13(7) (1992), 17-43.

9. O.A. Akinfenwa, S.N. Jator and N.M. Yao, Continuous block backward differentiation formula for solving stiff ordinary differential equations. Comput. Math. Appl., 65 (2013), 996-1005.
10. S. Ali, An approximation to solution of space and time fractional telegraph equations by He's variational iteration method. Math. Prob. Eng., (2010), ID-290631.
11. G. E. Andrews, R. Askey and R. Roy, Special Functions, vol. 71 of encyclopedia of mathematics and its applications. Cambridge University Press, Cambridge, UK 1999.
12. Arikoglu and I. Ozkol, Solution of fractional differential equations by using differential trans- form method. Chaos, Solitons Fract., 34 (2007), 1473-1481.