International Journal For Advanced Research In Science & Technology
A peer reviewed international journal www.ijarst.in
ISSN: 2457-0362

ANALYSING THE CONCEPT OF IDEMPOTENT MATRIX

B Jhansi Bala
Research Scholar Monda University, Delhi Hapur Road Village & Post Kastla, Kasmabad, Pilkhuwa, Uttar Pradesh
Dr. Rajeev Kumar
Research Supervisor Monda University, Delhi Hapur Road Village & Post Kastla, Kasmabad, Pilkhuwa, Uttar Pradesh

ABSTRACT
A matrix that remains unchanged when multiplied by itself is said to be idempotent. In regression analysis and the theory of linear statistical models, idempotent matrices are crucial, particularly in relation to the analysis of variance and the theory of least squares. The parameter k - idempotent is connected with and driven by the idea of k - idempotent matrices, which were first proposed by Krishnamoorthy et al. as a generalisation of idempotent matrices; in this paper, we present and investigate a new characteristic k - idempotent fuzzy matrix. A k-permuted version of an idempotent matrix is known as a k-idempotent matrix. The study's fundamental findings are presented. Also covered are the spectral and k-spectral theories of k-idempotent matrices.

Keywords: - Matrix, Idempotent, Probability, Torment.

I. INTRODUCTION

Since very early days, Man has an eagerness to know and predict the future though he knows it is uncertain. Uncertainty ranging from falling short of certainty to an almost complete lack of conviction or knowledge is attributed to many sources. One source of uncertainty is randomness which arises, for example when tossing a coin and not knowing which side of the coin will land up beforehand. Probability theory plays a vital role to capture uncertainty of a certain type that is due to randomness. Uncertainty is thus an important commodity in the modeling business, which can be traded for gains in the other essential characteristics of models. In science, Uncertainty is essential, it is not only an unavoidable torment, but it has, infact a great utility. Vagueness which results from imprecise information can take us to these different kinds of uncertainty.

In the later 19th century, the traditional view of uncertainty gradually started to change to the modern view. In 1937, American philosopher Max Black envisioned some ideas to handle the different types of uncertainty that are due to vagueness. But it is generally agreed that an important point in the evolution of the modern concept of uncertainty emerged after the publication of a seminar paper by Lotfi A Zadeh of the University of California at Berkeley in 1965. In his paper entitled Fuzzy sets, Zadeh introduced the theory as a generalization of crisp set, whose object fuzzy sets are sets with boundaries that are not precise. The membership in a fuzzy set is not a matter of affirmation of denial, but rather a matter of degree. It provides us not only with a meaningful representation of

International Journal For Advanced Research In Science & Technology
A peer reviewed international journal www.ijarst.in
IJARST ISSN: 2457-0362

measuring uncertainties, but also with a powerful representation of vague concepts expressed in natural language. After that, fuzzy set theory gaining a growing acceptability among Mathematicians, Engineers, Scientists and Philosophers.

II. PRELIMINARIES

Classical set

The set is a collection of well-defined objects, these objects are called element of a set.

$x \in A$ indicates that the object x belongs to the set A

Membership function

A function $\mu A(X)$ is said to be membership function if it belongs to the interval [0, 1].

Fuzzy set

Let A be the classical set and $\mu A(X)$ be the membership function, the fuzzy set is defined as $A* = \{(x, \mu A(X)) : x \in A, \mu A(X) \in [0, 1]$.

Operations on fuzzy set

The operations +, . and − on fuzzy values are defined as $a + b = \max\{a, b\}$, $a.b = \min\{a, b\}$ and $a - b =$

Fuzzy Matrix

A $n \times n$ matrix $A = [a_{ij}]$ with all $a_{ij} \in [0, 1]$ is called a fuzzy matrix, for fuzzy matrices $A = [a_{ij}]_{n\times n}$, $B = [b_{ij}]_{n\times p}$ and $C = [c_{ij}]_{n\times p}$, the following are defined :

Addition : $B + C = [b_{ij} + c_{ij}]$ where $b_{ij} + c_{ij} = \max\{b_{ij}, c_{ij}\}$

Multiplication : $AB = [a_{ij}]_{n\times n}[b_{ij}]_{n\times p}$

$$= \sum_{k=1}^{n} a_{ik}b_{kj} \text{ where } a_{ik}b_{kj} = \min\{a_{ik}, b_{kj}\}$$

Commutator

The commutator [A, B] of two matrices $A, B \in \mathcal{F}_n$ is defined as $[A, B] = AB - BA$.

Permutation matrix

A square matrix which contain exactly one '1' in each row and every column whether the other entries are '0' is called the permutation matrix.

Vector space

The set V_n together with the operations of component wise addition and fuzzy multiplication is called a fuzzy vector space.

Row space

The subspace of V_n spanned by the row vectors of $A \in \mathcal{F}_n$ is called the row space of A.

Column space

The subspace of V_n spanned by the column vectors of $A \in \mathcal{F}_n$ is called the column space of A.

Idempotent matrix

A square matrix $A \in \mathcal{F}_n$ is said to be idempotent if $A^2 \in \mathcal{F}_n$ and equals A.

Nilpotent

Let $A \in \mathcal{F}_n$ and $A^m = 0$ for $m \in N$, then A is said to be Nilpotent.

Quadrapotent

$A \in \mathcal{F}_n$ is said to be quadrapotent if $A^4 = A$

Transpose of A

If $A = [a_{ij}]$, then $A^T = [a_{ji}]$.

Adjoint of A

If $A = [a_{ij}]$, then $adjA = |A_{ji}|$, where the determinant $|A_{ji}|$is of $(n-1) \times (n-1)$ fuzzy matrix obtained by deleting row j and column i of A.

Determinant of A

Let $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ then the determinant of A is defined as $\det[A] = a_{21}a_{12} + a_{11}a_{22}$ under max−min composition.

Symmetric matrix

$A \in \mathcal{F}_n$ is called symmetric matrix if $A^T = A$.

Reflexive matrix

$A = [a_{ij}] \in \mathcal{F}_n$ is said to be reflexive if $A \geq I_n$, where I_n is the n × n identity matrix.

Transitive matrix

If $A = [a_{ij}] \in \mathcal{F}_n$ is said to be transitive if $A^2 \leq A$.

Compact matrix

If $A = [a_{ij}] \in \mathcal{F}_n$ and $A^2 \geq A$, then A is called a compact matrix.

Comparable matrices

$A \in \mathcal{F}_n$ and $B \in \mathcal{F}_n$ are said to be comparable matrices if either $A \leq B$ or $B \leq A$.

Invertible matrix

For A be the square fuzzy matrix, if $B \in \mathcal{F}_n$ exists such that $AB = BA = I_n$, then A is invertible.

Similaritry relation

A fuzzy relation R(X, Y) with the membership matrix MR is said to be similar if it is reflexive, symmetric and transitive.

That is $MR \geq I_n$, $MR = M_R^T$ and $M_R^2 \leq M_R$

III. CONCEPT OF IDEMPOTENT MATRIX

Idempotent matrix is a square matrix, which multiplied by itself, gives back the initial square matrix. A matrix M, when multiplied with itself, gives back the same matrix M, $M^2 = M$.

Let us consider a matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

Further since A is taken as an idempotent matrix, we have $A^2 = A$.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

$$\begin{pmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Here let us compare the terms on either sides.

$a^2 + bc = a$

$bc = a - a^2$

$ab + bd = b$

$ab + bd - b = 0$

$b(a + d - 1) = 0$

$b = 0$ or $a + d - 1 = 0$

$d = 1 - a$

From the above derivation we can understand that a matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is an idempotent matrix if $d = 1 - a$, and $bc = a - a^2$. Further using these two conditions for a 2 x 2 square matrix, we can create an idempotent matrix. Let us create an idempotent matrix by taking a = 5, and we have the other elements of the matrix as follows.

$d = 1 - a = 1 - 5 = -4$

$bc = a - a^2 = 5 - 5^2 = 5 - 25 = -20$

$bc = -20$

The possible combinations for the values of b and c are b = 10, and c = -2. Hence one of the idempotent matrices which can be formed is as follows.

$$P = \begin{pmatrix} 5 & 10 \\ -2 & -4 \end{pmatrix}$$

Also, all the identity matrices on multiplication with itself give back the identity matrix, and hence the identity matrix is also considered an idempotent matrix.

The determinant of an idempotent matrix is always equal to zero, and hence an idempotent matrix is also a singular matrix.

Definition:

Mathematically we can define Idempotent matrix as: A square matrix [A] will be called Idempotent matrix if and only if it satisfies the condition $A^2 = A$. Where A is n x n square matrix.

In other words, an Idempotent matrix is a square matrix which when multiplied by itself, gives result as same square matrix.

Also if square of any matrix gives same matrix (i.e, $A^2 = A$) then that matrix will be Idempotent matrix.

Here if we observe the definition $A^2 = A$, i.e, A = square of (A). It means we can say that the Idempotent matrix [A] is always the square of same matrix [A].

Conditions of Idempotent matrix

The necessary conditions for any 2 x 2 square matrix to be an Idempotent matrix is that either it should be diagonal matrix of order 2 x 2, or its trace value should be equal to **1**.

Application of Idempotent matrix

One of the very important applications of Idempotent matrix is that it is very easy and useful for solving [M] matrix and Hat matrix during regression analysis and econometrics. The idempotency of [M] matrix plays very important role in other calculations of regression analysis and econometrics.

It is very easy to check whether a given matrix [A] is an idempotent matrix or not. Simply multiply that given matrix [A] with same matrix [A] and find the square of given matrix [i.e, A^2] and then check that whether the square of matrix [A^2] gives resultant matrix as same matrix [A] or not, (i.e, $A^2 = A$). If this condition satisfies then given matrix will be idempotent matrix otherwise it will not be an idempotent matrix.

IV. CONCLUSION

The concept of k-idempotent matrices is introduced for complex matrices and exhibited as a generalization of k-idempotent matrices. It is shown that k-idempotent matrices are quadripotent. The conditions for power hermitian matrices to be k-idempotent are obtained. It is also proved that the set forms a group under matrix multiplication. It is shown that a k-idempotent matrix reduces to an idempotent matrix if and only if AK = KA. The diagonalizability of k-idempotent matrices is proved. Eigen values of a k-idempotent matrix are found to be 0, 1, ω andω^2. For a k-idempotent matrix A, relations between matrix functions tr A, rank A and det A are determined. It is found that the k-eigen values of a k - idempotent matrix are 0, 1 and -1. The spectral and k-spectral resolution of a k - idempotent matrix is also found. A list of necessary and sufficient conditions is given for a k -idempotent matrix to be an idempotent matrix. It is proved for a k-idempotent matrix that the following are equivalent:

(1) A is normal
(2) A is square hermitian
(3) KA is normal
(4) KA is hermitian

In this study, it is shown that all the standard partial orderings such as Lowener, star and rank subtractivity are preserved under the fixed product of disjoint transpositions k. That is all the partial orderings are preserved under k-unitary similarity. Relation between k-hermitian matrix and k-idempotent matrix is derived here by means of Lowener partial order. It is proved that all the partial orderings are preserved for k-idempotent matrices when they are squared.

REFERENCES

[1] Baksalary, Jerzy & Baksalary, Oskar & Styan, George. (2002). Idempotency of linear combinations of an idempotent matrix and a tripotent matrix. Linear Algebra and its Applications. 354. 21-34. 10.1016/S0024-3795(02)00343-9.

[2] Granat, Robert & Kagstrom, Bo & Kressner, Daniel. (2006). Reordering the Eigenvalues of a Periodic Matrix Pair with Applications in Control. Proceedings of the 2006 IEEE Conference on Computer Aided Control Systems Design, CACSD. 25 - 30. 10.1109/CACSD-CCA-ISIC.2006.4776619.

[3] Di Napoli, Edoardo & Kaplunovsky, Vadim. (2005). Unitary Matrix Model of a Chiral [SU(N)]^K Gauge Theory. Journal of High Energy Physics. 0510. 074. 10.1088/1126-6708/2005/10/074.

[4] Granat, Robert & Kagstrom, Bo & Kressner, Daniel. (2006). Reordering the Eigenvalues of a Periodic Matrix Pair with Applications in Control. Proceedings of the 2006 IEEE Conference on Computer Aided Control Systems Design, CACSD. 25 - 30. 10.1109/CACSD-CCA-ISIC.2006.4776619.

[5] Klopper, Rembrandt & Lubbe, Sam. (2007). The Matrix Method of Literature Review. Alternation. 14.

[6] Song, Seok-Zun & Kang, Kyung-Tae & Beasley, LeRoy. (2007). Idempotent matrix preservers over Boolean algebras. Reprinted from the Journal of the Korean Mathematical Society J. Korean Math. Soc. 44. 169-178. 10.4134/JKMS.2007.44.1.169.