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TRAVERSABILITY OF VICT GRAPHS AND VITACT GRAPHS

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ABSTRACT:

In this paper, we give characterizations of graphs whose vict graphs and vitact graphs are eulerian.

1. INTRODUCTION:

In 1990, Kulli and Muddebihal [2] introduced the concepts of the lict graph and lictact graph of a graph. In [2], the planarity, outerplanarity and minimally nonouterplanarity of these graph valued functions are discussed.

In 2016 , Muddebihal and Jayashree.B. Shetty [3]and [4] introduced the concepts of the vict graph and vitact graph of a graph. In [3]and [4], the planarity, outerplanarity and minimally nonouterplanarity of these graph valued functions are discussed.

We now study traversability of vict graphs and vitact graphs.

In this paper, we obtain characterizations of graphs whose vict graphs and vitact graphs are eulerían.

A cutpoint of a connected graph G is one whose removal increases the number of components and a bridge is such a line.

By the parity of point we mean the parity of its degree.

The points and lines of a graph are called its members.

The vict graph $V_n(G)$ of a graph G is the graph whose point set is the union of the set of points and the set of cutpoints of G in which two points are adjacent if and only if the corresponding members of G are adjacent or the corresponding members of G are incident. The vitact graph $V_n(G)$ of a graph G is the graph whose point set is the union of the set of points and the set of cutpoints of G in which two points are adjacent if and only if the corresponding members of G are adjacent or incident.

The following definitions will be noted for later use

THEOREM 7.A.[1]. Let G be a connected graph. Then L(G) is eulerian if and only if the degrees of all the points of G are of the same parity.

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THEOREM 7.B.[2]. Let G be a connected graph, then L(G) = n(G)= $V_n(G)$ if and only if G is a block.

THEOREM 7.C.[2]. The vict graph $V_n(G)$ and vitact graph $V_m(G)$ of a graph G are isomorphic if and only if G has at most one cut point

The following are the simple Lemmas which we merely state.

LEMMA 7.1. If a point v of $V_n(G)$ corresponds to a cutpoint C of a graph G ,deg r(G) v = r , where r is the number of vertices incident to C

LEMMA 7.2.Let in $V_n(G)$ the point u correspond to a cutpoint x of a graph G. Then deg $V_n(G)$ u = n_1+ n_2 where n_1 is the number of vertices adjacent with x and n_2 is the number of cutpoints incident to x.

LEMMA 7.3. If a point v of $V_m(G)$ corresponds to a vertex of a graph G, then deg $V_m(G)$ v = deg $V_n(G)$ v .

LEMMA 7.4. If a point v of $V_m(G)$ corresponds to a cutpoint C of a graph G, deg $V_m(G)$ v = deg $V_n(G)$ v+n where n is the number of cutpoints adjacent to C.

2. EULERIAN GRAPHS

In the following theorem, we give a characterization of graph whose vict graph is eulerian.

THEOREM 1. The vict graph $V_n(G)$ of a connected graph G is eulerian if and only if

(1) G is a block and the degrees of all the points of G are of the same parity.

or (2) if x is a line of G, then the number of vertices adjacent with x and the number of cutpoints incident to x are either all even or odd.

PROOF. Suppose $V_n(G)$ is eulerian. We consider the following cases.

Case 1. Suppose G is a block and not all points of G are of the same parity. Then by Theorem 7.A, L(G) is not eulerian. By Theorem 7.B. n(G) is not eulerian, a contradiction. Thus (1) holds.

Case 2. Suppose x = v_1 v_2 is a line of G. Then by Lemma 7.2.

deg $V_n(G)$ x =n_1 + n_2

where n_1 is the number of vertices adjacent with x and n_2 is the number of cutpoints incident to x. Since deg $V_n(G)$ is even, n_1 and n_2 are both even or odd. In both cases (2) holds

Conversely, suppose the condition (1) on G holds. Then by Theorem 7.A. L(G) is eulerian and by Theorem 7.B, $V_n(G)$ is eulerian. Suppose the condition (2) on G holds. Assume x is a point of $V_n(G)$. Then x is a point or a cutpoint of G. If x is a cutpoint of G. then by Lemma 7.1. deg $V_n(G)$ x = r where r is the number of vertices incident to x. Clearly deg $V_n(G)$ x is even. If x is a point of G, then by the condition (2) deg $V_n(G)$ x is even and $V_n(G)$ is eulerian This completes the proof.

A necessary and sufficient condition for a graph whose vitact graph is eulerian is presented in the following theorem.

THEOREM 2. The vitact graph $V_m(G)$ of a connected graph G is eulerian if and only if

(1) G is a block and the degrees of all the points of G are of the same parity.
or (2) if x is a point of G, then the number of vertices adjacent with x and the number of cutpoints incident to x are either all even or odd.
and (3) every cutpoint of G is adjacent with an even number of cutpoints

PROOF. Suppose $V_m(G)$ is eulerian . Assume G is a block and not all points of G are of the same parity. Then by Theorem 1, . $V_n(G)$ is not eulerian. By Theorem 7.C, $V_m(G)$ is not eulerian. Thus (1) holds.
Now we prove the condition (2). By Theorem 7.1, one can easily see that $V_n(G)$ is eulerian. Hence the condition (2) holds.
Lastly, suppose a point v of $V_m(G)$ corresponds to a cutpoint u of G. Now since $V_n(G)$ is eulerian, deg $V_n(G)$ v is even.But deg $V_m(G)$ v is also even. By Lemma 7.4, u is adjacent with an even number of cutpoints of G.
Conversely, suppose the condition (1) on G holds. Then by Theorem 7.1. n(G) is eulerian and by Theorem 7.C, $V_m(G)$ is eulerian.
Suppose the conditions (2) and (3) on G hold. By Theorem 1, condition (2) implies that n(G) is eulerian and hence for any point v. deg $V_n(G)$ v is even.
Consider the following two cases.
Case 1. Let v correspond to a line of G. Then by Lemma 3,
deg $V_m(G)$ v = deg $V_n(G)$ v.
Case 2. Let v correspond to a cutpoint C of G. Then by Lemma 4, deg $V_m(G)$ v = deg $V_n(G)$ v + n, where n is the number of cutpoints adjacent to C. But by condition (3), n is even. Hence the degree of v in $V_m(G)$ in either case is even. Thus $V_m(G)$ is eulerian This completes the proof.

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